

Krein-Mil'man theorem after Prof. Tagamlitzki

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Abstract

In this talk we briefly survey the beautiful theory which has arisen out of the investigations of the Krein-Mil'man property and generalizations of the concept of extreme point: strongly extreme point, denting point, exposed point, strongly exposed point etc.

It is said that a Banach space X has the Krein-Mil'man property if each closed bounded convex subset of X is the norm closed convex hull of its extreme points. The interrelation between the Krein-Mil'man property and the Radon-Nikodým property was studied by some of the most prominent twentieth century mathematicians (e.g. J. Bourgain, M. Talagrand, W. Schachermayer and others). It is amazing that Radon-Nikodým property possesses characterizations of very different nature: they belong to Geometry of Banach spaces, Vector Measures Theory, Optimization (theory of Asplund spaces), Variational Analysis (Stegall's variational principle). Parts of this theory already belong to textbooks.

It is known that the extremal phenomena were one of the centers of the research of Prof. Tagamlitzki (c.f., e.g., [11], [12], [13]). The history of the development of this theory and the history of the Department of Mathematical Analysis of FMI-SU are closely interwoven and they are worth to be known to the younger generation of Bulgarian mathematicians.

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