Fast Converging Sequence To Euler-Mascheroni Constant

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Abstract

The usual definition of the Euler-Mascheroni constant γ is the equality

$$\gamma = \lim_{n\to\infty}\left(1+\frac{1}{2}+...+\frac{1}{n}-\ln n\right).$$

Since this sequence converges to γ very slowly, it is not suitable for the effective computation of γ. This is why other much faster methods are invented, like the method of Karatsuba in [2], which is suitable to prove polynomial-time computability of γ.

The aim of this paper is to study the complexity of γ in another context, namely the subrecursive class $\mathcal{M}^2$, contained in the third level $\mathcal{E}^2$ of Grzegorczyk's hierarchy. It appears that the known methods for the effective computation of γ are not suitable in this context.

The author has proven in [1] that γ is $\mathcal{M}^2$-computable. This is done by applying some fast convergence properties of the trapezoidal rule, described in [3], to an integral representation of γ. In the present paper, an actual sequence is extracted from this proof, which converges to γ with subexponential convergence rate.

Keywords: computable real number, subrecursive complexity, Euler-Mascheroni constant, exponential trapezoidal rule

References

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