Exact behavior of singular solutions to (3+1)-D Protter-Morawetz problem for Keldysh-type equations

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Abstract

We study a four-dimensional boundary value problem (BVP) for hyperbolic - parabolic equations of Keldysh type. Similar problem for weakly hyperbolic equations of Tricomi type was proposed by M. Protter as multidimensional analogue of the Darboux problem on the plane. It is well known that the setting and the study of BVPs for equations of Keldysh-type are always more difficult and complicated. Recently, there has been a lot of interest in the regularity of solutions of various BVPs for linear and nonlinear Keldysh-type equations and their specific applications in plasma physics, optics, and analysis on projective spaces (see [1]). In [2] and [3] we formulate a (3+1)-D Protter-Morawetz problem for equations of Keldysh type and find new effects that appear in this more delicate case. Unlike the Tricomi case, in this problem, a data on the degenerate boundary is not prescribed (similarly to the elliptic-parabolic case) and the normal derivative of the solution can have singularity on it, but up to a certain level. It is shown that in the frame of classical solvability the considered problem is not Fredholm, because it has infinite-dimensional co-kernel. Alternatively, a notion of a generalized solution with possible singularity is given. It is interesting that this singularity is isolated at only one boundary point, which makes this case different from the traditional case of propagation of singularity. An exact integral representation of the generalized solution is found and the existence and uniqueness of such solution are proved. Further, the asymptotic behavior of the generalized solutions at the singular point is found. There are presented some orthogonality conditions on the right-hand side functions, which are necessary and sufficient for existence of generalized solutions with fixed order of singularity.

REFERENCES

[1] T. Otway, *The Dirichlet Problem for Elliptic-Hyperbolic Equations of Keldysh Type*, Lecture Notes in Mathematics, vol. 2043, Springer - Verlag Berlin Heidelberg (2012).
[2] N. Popivanov, T. Hristov, A. Nikolov, M. Schneider, *On the existence and uniqueness of a generalized solution of the Protter problem for (3+1)-D Keldysh-type equations*, Boundary Value Problems, 2017 (2017), art. no. 26, 30 p., DOI 10.1186/s13661-017-0757-1.
[3] N. Popivanov, T. Hristov, A. Nikolov, M. Schneider, *Singular solutions to a (3+1)-D Protter-Morawetz problem for Keldysh-type equations*, Advances in Mathematical Physics, (2017), 24 p. (in print)