Lower bounding the Folkman numbers $F_v(a_1, ..., a_s; m-1)$

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Abstract

For a graph G the expression $G \overset{v}{\rightarrow} (a_1, ..., a_s)$ means that for every s-coloring of the vertices of G there exists $i \in \{1, ..., s\}$ such that there is a monochromatic a_i-clique of color i. The vertex Folkman numbers

$$F_v(a_1, ..., a_s; m-1) = \min\{|V(G)| : G \overset{v}{\rightarrow} (a_1, ..., a_s) \text{ and } K_{m-1} \not\subseteq G\}.$$

are considered, where $m = \sum_{i=1}^{s}(a_i - 1) + 1$. We know the exact values of all the numbers $F_v(a_1, ..., a_s; m-1)$ when $\max\{a_1, ..., a_s\} \leq 6$ and also the number $F_v(2, 2, 7; 8) = 20$. In this paper we present a method for obtaining lower bounds on these numbers. In the special case when $\max\{a_1, ..., a_s\} = 7$ we prove that $F_v(a_1, ..., a_s; m-1) \geq m + 11$ and this bound is exact for all m. The known upper bound for these numbers is $m + 12$.