Estimates for the best constant in a Markov L_2-inequality with the assistance of computer algebra

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Abstract

We prove two-sided estimates for the best (i.e., the smallest possible) constant $c_n(\alpha)$ in the Markov inequality

$$\|p_n'\|_{w_\alpha} \le c_n(\alpha)\|p_n\|_{w_\alpha}\,, \qquad p_n \in \mathcal{P}_n\,.$$

Here, $\mathcal{P}_n$ stands for the set of algebraic polynomials of degree at most n, $w_\alpha(t) := t^\alpha\, e^{-t}$, $\alpha > -1$, is the Laguerre weight function, and $\|\cdot\|_{w_\alpha}$ is the associated L_2-norm,

$$\|f\|_{w_\alpha} = \left(\int_0^\infty |f(x)|^2 w_\alpha(x)\, dx\right)^{1/2}\,.$$

Our approach is based on the fact that $c_n^{-2}(\alpha)$ equals to the smallest zero of the $n^{\rm th}$ degree polynomial Q_n in the sequence of polynomials orthogonal with respect to a measure supported on $[0,\infty)$ and defined by an explicit three-term recurrence relation. We employ computer algebra to evaluate the seven lowest degree coefficients of Q_n and to obtain thereby bounds for $c_n(\alpha)$. This work is a continuation of a recent paper by Nikolov and Shadrin (2017), where estimates for $c_n(\alpha)$ were proven on the basis of the four lowest degree coefficients of Q_n.