Pohozhaev identities as conservation laws for semi-linear elliptic-hyperbolic equations

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Абстракт:

It is well known result of Pohozhaev (1965), that the homogeneous Dirichlet problem for semilinear elliptic equations $\Delta u + u|u|^{p-2} = 0$, in a bounded subset Ω of R^n, with $n > 2$, permits only the trivial solution if the domain is star-shaped, the solution is sufficiently regular, and the power of nonlinearity $p > 2^*(n) := 2n/(n-2)$, where the latter quantity is the critical exponent in the Sobolev embedding of $H_0^1(\Omega)$ into $L^p(\Omega)$ for $p < 2^*(n)$. To the opposite of this fact, in the case $2 < p < 2^*(n)$ there exist nontrivial solutions. In the last 50 years the Pohozhaev identities and results have been used and extended for a large class of elliptic problems. Let us mention now that in [1], [2] it has been shown that the nonexistence principle in supercritical case also holds for certain two dimensional problems for the mixed elliptic-hyperbolic Gellersted operator L (instead of Δ), with some appropriate boundary conditions. It is also valid for a large class of such problems even in higher dimensions [3]. In dimension 2, such operators have a long-standing connection with transonic fluid flow. Of course, the critical Sobolev embedding in this case is for a suitable weighted version of $H_0^1(\Omega)$ into $L^p(\Omega)$. As usual, in the BVP for such mixed elliptic-hyperbolic Gellersted operator L, the boundary data are given only on the proper subset of the boundary of Ω. To compensate the lack of a boundary condition on a part of boundary, a sharp Hardy-Sobolev inequality is used, as was first done in [1], [2] and later in [3], [4]. Some further results, already published or in progress, prepared jointly with colleagues from Italy and Norway will be also discussed.

REFERENCES

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