On the Largest Critical Value of $T_n^{(k)}$

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Abstract

Let $T_n(x) = \cos n \arccos x,\ x \in [-1,1]$, be the n-th Chebyshev polynomial of the first kind and, for $1 \le k \le n-2$, let $\omega_{n,k}$ be the rightmost zero of $T_n^{(k+1)}$. Here we study the quantity

$$\tau_{n,k} := \frac{|T_n^{(k)}(\omega_{n,k})|}{T_n^{(k)}(1)}\,.$$

We show that the sequence $\{\tau_{n,k}\}_{n=k+2}^{\infty}$ is monotonically decreasing and obtain both upper bounds and asymptotic formulae for the quantities

$$\tau_k^* := \lim_{n\to\infty} \tau_{n,k} \quad \text{and} \quad \tau_{k+m,k}\,,\ m \ge 2\,.$$