Markov-type inequality in the L_2-norm induced by the Gegenbauer weight

Geno Nikolov*, Alexei Shadrin†
*Faculty of Mathematics and Informatics
Sofia University "St. Kliment Ohridski"
geno@fmi.uni-sofia.bg.
†Department of Applied Mathematics and Theoretical Physics
Cambridge University, United Kingdom

Abstract

Let $w_\lambda(t) := (1-t^2)^{\lambda-1/2}$, where $\lambda > -\frac{1}{2}$, be the Gegenbauer weight function, let $\|\cdot\|_{w_\lambda}$ be the associated L_2-norm,

$$\|f\|_{w_\lambda} = \left\{ \int_{-1}^{1} |f(x)|^2 w_\lambda(x)\,dx \right\}^{1/2},$$

and denote by $\mathcal{P}_n$ the space of algebraic polynomials of degree $\le n$. We study the best constant $c_n(\lambda)$ in the Markov inequality in this norm

$$\|p_n'\|_{w_\lambda} \le c_n(\lambda)\|p_n\|_{w_\lambda}, \qquad p_n \in \mathcal{P}_n,$$

namely the constant

$$c_n(\lambda) := \sup_{p_n \in \mathcal{P}_n} \frac{\|p_n'\|_{w_\lambda}}{\|p_n\|_{w_\lambda}}.$$

We derive explicit lower and upper bounds for the best Markov constant $c_n(\lambda)$,

$$\underline{c}_n(\lambda) \le c_n(\lambda) \le \overline{c}_n(\lambda)$$

with a small ratio $\dfrac{\overline{c}_n(\lambda)}{\underline{c}_n(\lambda)}$, which are valid for all $n \in \mathbb{N}$ and $\lambda > -1/2$.

REFERENCES

[1] G. Nikolov, A. Shadrin, On the Markov inequality in the L_2–norm with the Gegenbauer weight, Constr. Approx., to appear.