Definite quadrature formulae of order 3 with equidistant nodes

Ana Avdzhieva, Vesselin Gushev, Geno Nikolov
Faculty of Mathematics and Informatics
Sofia University "St. Kliment Ohridski"
aavdzhieva@fmi.uni-sofia.bg.

Abstract

The compound rectangles and trapezium quadrature formulae are classical examples of positive (respectively, negative) definite quadrature formulae of order 2. Besides providing lower and upper bounds for definite integrals of convex or concave (i.e., 1–monotone) integrands, they are convenient from computational point of view as they use equidistant nodes.

More generally, all Newton–Cotes quadrature formulae are definite of even order (however, since (roughly) half of their coefficients are negative, higher order Newton-Cotes quadratures are not used in practice). G. Schmeisser [1] constructed definite quadrature formulae of even order which use equidistant nodes and are asymptotically optimal. In [2] the authors constructed asymptotically optimal definite quadrature formulae of order 4 with almost equidistant nodes, and proved a posteriori error estimates for some pairs of such quadratures.

Unlike the definite quadrature formulae of even order, definite quadrature formulae of odd order are never symmetric. Somewhat unexpectedly, this phenomenon turns out to be an advantage rather than disadvantage. For instance, when reflecting the nodes of a positive definite quadrature formula of odd order (keeping the weights unchanged), we obtain a negative definite quadrature formula and vice versa. This allows derivation of simple a-posteriori error estimates, i.e. estimates that do not require knowledge of the magnitude of any derivative but just few evaluations of the integrand.

We construct sequences of definite quadrature formulae of order 3 based on equidistant nodes, i.e., they use the nodes of either the rectangles or the trapezium quadrature formulae and, excluding the coefficients of the boundary three or four nodes, have the same coefficients. A kind of opitmization is performed for the choice of the boundary coefficients so that the error constants of constructed quadratures are as small in absolute value as possible.

REFERENCES

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[2] A. Avdzhieva, G. Nikolov, Asymptotically optimal quadrature formulae of 4^{th} order, J. Comp. Appl. Math. 311 (2017) 565–582.